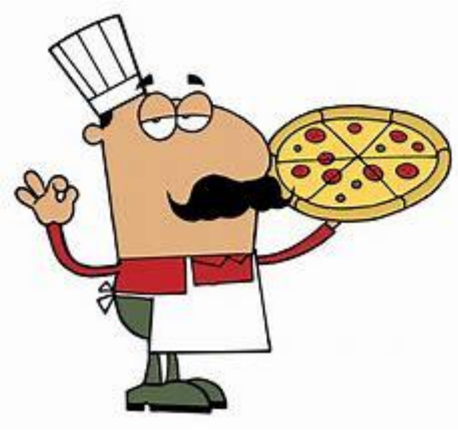
ADDING AND SUBTRACTING FRACTIONS – PART II

We continue our analysis of algebraic fractions. Now the fractions will have more stuff in their numerators and denominators, but the basic rule still applies: To **add** or **subtract** fractions, we <u>must</u> have a common denominator.



❑ *MORE ADDING AND SUBTRACTING*

<u>EXAMPLE 1</u>: **Subtract:** $\dfrac{\mathbf{7}}{\boldsymbol{x+3}} - \dfrac{\mathbf{3}}{\boldsymbol{x-2}}$

<u>Solution</u>: The two denominators have no factor in common. (Yes, they both have an x, but this is a term, not a factor.) The LCD is the product of the two denominators: $(x + 3)(x - 2)$. So we'll multiply the top and bottom of the first fraction by $(x - 2)$, and multiply the top and bottom of the second fraction by $(x + 3)$:

$$\frac{7}{x+3} - \frac{3}{x-2} \qquad \text{(the original problem)}$$

$$= \frac{7}{x+3}\underbrace{\left[\frac{x-2}{x-2}\right]}_{=1} - \frac{3}{x-2}\underbrace{\left[\frac{x+3}{x+3}\right]}_{=1} \qquad \text{(build up to the LCD)}$$

$$= \quad \frac{7(x-2)}{(x+3)(x-2)} - \frac{3(x+3)}{(x-2)(x+3)} \qquad \text{(the bottoms are the same)}$$

Same denominator!

$$= \quad \frac{7(x-2)-3(x+3)}{(x-2)(x+3)} \qquad \text{(combine the two fractions)}$$

$$= \quad \frac{7x-14-3x-9}{x^2+x-6} \qquad \text{(distribute)}$$

$$= \quad \boxed{\frac{4x-23}{x^2+x-6}} \qquad \text{(combine like terms)}$$

<u>EXAMPLE 2:</u> **Add:** $\mathbf{\dfrac{5}{m^2+5m+6} + \dfrac{3}{m+2}}$

<u>Solution:</u> This is a tricky one. To add these fractions, we have to multiply tops and bottoms by factors that will result in the LCD. But to know what factors to multiply by, we have to know what factors are already in the denominators. So, we factor the denominators:

$$\frac{5}{m^2+5m+6} + \frac{3}{m+2} = \frac{5}{(m+2)(m+3)} + \frac{3}{m+2}$$

Now we can easily see the factors in the denominators. To make the denominators the same, we need to multiply the top and bottom of the second fraction by $m + 3$. Then we can add the fractions:

$$\frac{5}{(m+2)(m+3)} + \frac{3}{m+2}\left[\frac{m+3}{m+3}\right] = \frac{5}{(m+2)(m+3)} + \frac{3(m+3)}{(m+2)(m+3)}$$

$$= \quad \frac{5+3(m+3)}{(m+2)(m+3)} = \frac{5+3m+9}{m^2+5m+6} = \boxed{\frac{3m+14}{m^2+5m+6}}$$

<u>EXAMPLE 3:</u> **Subtract:** $\dfrac{\mathbf{8}}{\boldsymbol{u^2+3u-4}} - \dfrac{\boldsymbol{7-5u}}{\boldsymbol{u^2-3u-28}}$

<u>Solution:</u> First factor all the denominators:

$$\frac{8}{(u-1)(u+4)} - \frac{7-5u}{(u+4)(u-7)}$$

Now build up the fractions so that the LCD is created:

$$= \frac{8}{(u-1)(u+4)}\left[\frac{u-7}{u-7}\right] - \frac{7-5u}{(u+4)(u-7)}\left[\frac{u-1}{u-1}\right]$$

$$= \frac{8(u-7)}{(u-1)(u+4)(u-7)} - \frac{(7-5u)(u-1)}{(u-4)(u-7)(u-1)}$$

Notice that the denominators are the same.

$$= \frac{8(u-7) \;-\; (7-5u)(u-1)}{(u-1)(u+4)(u-7)}$$

(subtract the numerators)

Notice the parentheses.

$$= \frac{8u-56-(7u-7-5u^2+5u)}{(u-1)(u+4)(u-7)}$$

(distribute)

$$= \frac{8u-56-(-5u^2+12u-7)}{(u-1)(u+4)(u-7)}$$

(combine like terms)

$$= \frac{8u-56+5u^2-12u+7}{(u-1)(u+4)(u-7)}$$

(distribute the minus sign)

$$= \boxed{\frac{5u^2-4u-49}{(u-1)(u+4)(u-7)}}$$

(combine like terms)

You may wonder why we left the denominator in factored form. It's just a matter of opinion, but many teachers figure that the problem's hard enough without multiplying out those three binomials.

Homework

1. Perform the indicated operation:

a. $\frac{6}{d+1} + \frac{-4}{6d+5}$

b. $\frac{-6}{5h-2} - \frac{5}{4h-1}$

c. $\frac{-6}{q+1} + \frac{3}{5q+4}$

d. $\frac{3}{2t+5} - \frac{-6}{2t+1}$

2. Perform the indicated operation:

a. $\frac{3}{z-3} + \frac{1}{z^2+12z+32}$

b. $\frac{-3}{u^2+u-72} - \frac{3}{u+9}$

c. $\frac{-2}{w^2-19w+90} - \frac{2}{w-10}$

d. $\frac{4}{h+7} + \frac{-5}{h^2+11h+28}$

3. Perform the indicated operation:

a. $\frac{7}{b^2+5b-6} + \frac{3b-1}{b^2-4b-60}$

b. $\frac{-6}{a^2-16} - \frac{-5a+3}{a^2-5a+4}$

c. $\frac{-1}{s^2-s-42} - \frac{6s-7}{s^2-4s-21}$

d. $\frac{9}{v^2-9} + \frac{2v+1}{v^2+13v+30}$

Solutions

1. a. $\dfrac{32d+26}{(d+1)(6d+5)}$ or $\dfrac{32d+26}{6d^2+11d+5}$

b. $\dfrac{-49h+16}{(5h-2)(4h-1)}$

c. $\dfrac{-27q-21}{(q+1)(5q+4)}$

d. $\dfrac{18t+33}{(2t+5)(2t+1)}$

2. a. $\dfrac{3z^2+37z+93}{(z+4)(z+8)(z-3)}$

b. $\dfrac{-3u+21}{(u+9)(u-8)}$

c. $\dfrac{-2w+16}{(w-10)(w-9)}$

d. $\dfrac{4h+11}{(h+7)(h+4)}$

3. a. $\dfrac{3b^2+3b-69}{(b-1)(b+6)(b-10)}$

b. $\dfrac{5a^2+11a-6}{(a+4)(a-4)(a-1)}$

c. $\dfrac{-6s^2-30s+39}{(s+6)(s-7)(s+3)}$

d. $\dfrac{2v^2+4v+87}{(v-3)(v+3)(v+10)}$

"The illiterate of the 21st century will not be those who cannot read and write, but those who cannot learn, unlearn, and relearn."

– Alvin Toffler